Volume of Solids

Dear Family,

In this topic, your child is learning about volume and solids. First of all, your child is learning to describe solids in exact mathematical terms. For example, a three-dimensional box shape is a rectangular prism. A pipe or tube shape is a cylinder. An Egyptian pyramid is a rectangular pyramid (there are also triangular pyramids, with a triangular base). The parts of solids also have special names. A flat polygon-shaped surface of a solid is called a face. The line segments where two faces meet is an edge, and the point where three or more edges meet is a vertex.

Here is an activity you can do to help your child become familiar with solids and their properties.

Solid Information

![Solid icons: Sphere, Triangular Prism, Rectangular Prism, Cylinder, Triangular Pyramid, Rectangular Pyramid, Cone]

**Step 1:** Talk with your child about the properties of the solids on this page. How many faces does each solid have? How many bases? How many edges? How many vertices? Which solids have faces that are triangles? Which solids have faces that are rectangles? Which solids have faces that are circles? Which solid has only one face?

**Step 2:** Try to find an item in the shape of each solid. Encourage your child to relate a household object to each shape of each solid. If any shapes have yet to be related, ask your child to think of an object that might be found elsewhere or is a part of a building or a part of a machine.
three-dimensional shape (solid)

A solid figure that takes up space

draw here

edge

A line segment at which two faces meet in a solid

cube

A solid with six flat surfaces called faces. All of the faces are squares
**face**
A flat polygon-shaped surface of a polyhedron

**vertex (of a solid)**
The point at which three or more edges meet in a solid. The plural form of vertex is vertices

**prism**
A solid with two parallel bases that are the same size and same shape, and faces that are parallelograms
cylinder

A solid with two circular bases that are the same size, the same shape, and parallel.

pyramid

A solid with a base that is a polygon and whose other faces are triangles with a common vertex.

cubic unit

The volume of a cube 1 unit on each edge.

cubic units
cone
A solid with one circular base; the points on the circle are joined to one point outside the base.

volume
The number of cubic units needed to fill a solid figure.
Solids

A vertex of a solid is the point at which three or more edges meet.
An edge of a solid is a line segment where two faces meet.
A face of a solid is a flat polygon-shaped surface.

Here are some common three-dimensional shapes:
- Cube
- Rectangular Prism
- Cylinder
- Cone

For 1 through 3, tell which solid each object resembles.
1. Rectangular prism
2. Cylinder
3. Cone

In 4 through 6, complete each table by writing the number of vertices, edges, and faces in each solid shown at the right of each table.

<table>
<thead>
<tr>
<th>4.</th>
<th>Vertices</th>
<th>Edges</th>
<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5.</th>
<th>Vertices</th>
<th>Edges</th>
<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6</td>
<td>9</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6.</th>
<th>Vertices</th>
<th>Edges</th>
<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

For 4 and 5, complete each table by writing the number of vertices, edges, and faces in each solid shown at the right of each table.

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<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>5.</th>
<th>Vertices</th>
<th>Edges</th>
<th>Faces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
<td>15</td>
<td>7</td>
</tr>
</tbody>
</table>

6. What is the name of the three-dimensional shape at the right?
   - A. Cone
   - B. Triangular prism
   - C. Pyramid
   - D. Rectangular prism

7. How many vertices does a cone have? Explain.
   **1:** The circular base does not have any vertices but every point on the circle meets at one point, the vertex, outside the base.
Views of Solids

Here is what each figure on the left would look like from the front, side, and top. The number of cubes that can't be seen for each figure is written.

1. All the cubes are visible.
2. All the cubes are visible.
3. One of the cubes is not visible.

Look at the figure. Label its front, side, and top views.

1. Side
2. Front
3. Top
4. None
5. One
6. None

For 1 and 2, draw front, side, and top views of each stack of unit blocks.

1.

2.

3. In the figure for Exercise 2, how many blocks are not visible?
1

4. In the figure at the right, how many unit blocks are being used?
A 6
B 7
C 10
D 11

5. A figure is made from 8 unit blocks. It is 3 units tall. What is the maximum length the figure could be? Explain.

The maximum length is 6, because 2 are used to give the figure height.
Problem Solving: Use Objects and Solve a Simpler Problem

At a math fair, Willie saw a puzzle about a giant cube made of smaller identical white cubes. The giant cube was $4 \times 4 \times 4$. It contained 64 smaller cubes. Each of the six faces of the giant cube was painted red. The puzzle asked, “If the giant cube were taken apart, how many smaller cubes would have only one face painted red?” Here is how Willie tried to solve the puzzle.

1. Construct Cube A using 8 smaller cubes and Cube B using 27 smaller cubes. Imagine painting Cubes A and B.

2. Classify the smaller cubes. Think: Where are the cubes located in the Cubes A and B? How are they painted differently from each other? Make a table to organize the data.

<table>
<thead>
<tr>
<th>Cube A</th>
<th>Cube B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Number</td>
</tr>
<tr>
<td>Corner</td>
<td>8</td>
</tr>
<tr>
<td>Edge</td>
<td>none</td>
</tr>
<tr>
<td>Face</td>
<td>none</td>
</tr>
<tr>
<td>Center</td>
<td>none</td>
</tr>
</tbody>
</table>

Willie organized the data about the 84 smaller cubes in the giant cube. Use the table above to complete the table below. One set of data has already been completed.

<table>
<thead>
<tr>
<th>Giant Cube</th>
<th>Location</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Corner (Three sides on a $3 \times 3 \times 3$)</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>Edge (Think: One more than a $3 \times 3 \times 3$ on each edge)</td>
<td>24</td>
</tr>
<tr>
<td>1</td>
<td>Face (Think: Three more than a $3 \times 3 \times 3$ on each face)</td>
<td>24</td>
</tr>
<tr>
<td>0</td>
<td>Center (Think: The center is none)</td>
<td>8</td>
</tr>
</tbody>
</table>

1. Six people can be seated at a table. If two tables are put together, 10 people can be seated. How many tables are needed to make a long table that will seat 22 people?

   5 tables

2. A large cube has 5 layers, each with 5 rows of 5 small cubes. How many small cubes will the larger cube contain?

   $5 \times 5 \times 5 = 125$

3. There are 5 kinds of fish that Jerome feeds: guppies, zebra danios, bettas, platies, and neon tetras. Use the following clues to find the order in which Jerome feeds them:

   - Jerome feeds the guppies first.
   - Jerome does not feed the bettas right before or right after the guppies.
   - Jerome feeds the zebra danios last.
   - Jerome feeds the platys after the bettas.

   A | Guppies, zebra danios, bettas, platys, and neon tetras
   B | Bettas, platys, guppies, neon tetras, zebra danios
   C | Neon tetras, zebra danios, guppies, platys, bettas
   D | Bettas, guppies, platys, neon tetras, zebra danios

4. Suppose Ann is placing bowling pins in the following manner: 1 pin in the first row, 2 pins in the second row, 3 pins in the third row, and so on. How many pins will she use if she has 6 rows in her placement? Explain.

   $1 + 2 + 3 + 4 + 5 = 15$; Each row has 1 more than the previous row.
Models and Volume

Volume is the measure of space inside a solid figure. If you had a box, the number of cubic units it would take to fill the box would be the volume.

Find the volume of this box in cubic units by counting the number of unit cubes.

There are 16 cubes in the front layer and there are two layers. The total number of unit cubes is 32.

So, the volume is 32 cubic units.

In 1 through 6, find the volume in cubic units.

1. 24 cubic units
2. 16 cubic units
3. 27 cubic units
4. 20 cubic units
5. 36 cubic units
6. 48 cubic units

7. Draw a solid figure that has a volume of 10 cubic units.

Sample drawing:

8. A jewelry store received a package of rings that is 16 inches long, 10 inches wide, and 12 inches high. The package contains 1-inch cubes that each hold one ring. How many rings did the jewelry store receive? Explain how you found your answer.

1,920 rings; Sample answer: I found the volume of the package. 16 \times 10 \times 12 = 1,920 cubic inches. Each box holding 1 ring is 1 cubic inch. So, there are 1,920 rings.

Practice

Models and Volume

Find the number of cubes needed to make each rectangular prism. You can use unit cubes or you can count the cubes by looking at the drawing.

1. 60
2. 96
3. 24
4. 192
5. 48
6. 15

7. In the space below, draw a model of a rectangular prism 5 cubes long x 4 cubes wide x 2 cubes high.

8. How many cubes would it take to make a model of a rectangular prism that is 3 units long x 2 units wide x 4 units high?

A. 48  B. 24  C. 12  D. 0

9. How can you find the volume of a rectangular prism using a model?

Sample answer: If you have a model of the prism, you can count the unit cubes. You can also multiply the number of cubes in the width by the number of cubes in the height and then by the number of cubes in the length.
Volume

Volume is a measure of the space inside a solid figure. It is measured in cubic units. A cubic unit is the volume of a cube that has edges that are each 1 unit.

How to find the volume of a rectangular prism

Counting unit cubes:

Using a formula:

You know the length l, the width w, and the height h. Calculate the volume V using the formula

\[ V = l \times w \times h \]

Count the cubes in each layer; 6 cubes.
Multiply by the number of layers.

6 cubes \times 3 = 18 cubes
The volume of each cube is 1 cm\(^3\).
The volume of the prism is 24 cm\(^3\).

Find the volume of each rectangular prism using a formula.

1.

\[ V = l \times w \times h \]

\[ 8 \text{ m} \times 4 \text{ m} \times 3 \text{ m} = 8 \text{ m}^3 \]

2.

\[ V = l \times w \times h \]

\[ 20 \text{ ft} \times 12 \text{ ft} \times 8 \text{ ft} = 1,920 \text{ ft}^3 \]

Find the volume of each rectangular prism.

1. base area 56 in\(^2\), height 6 in. \(336 \text{ in}^3\)
2. base area 28 cm\(^2\), height 12 cm \(336 \text{ cm}^3\)
3. base area 42 m\(^2\), height 8 m \(336 \text{ m}^3\)

4. \(125 \text{ yd}^3\)
5. \(160 \text{ cm}^3\)
6. What is the volume of the cereal box? \(312 \text{ in}^3\)
7. What is the volume of this solid?
   A. \(32 \text{ m}^3\)
   B. \(22 \text{ m}^3\)
   C. \(200 \text{ m}^3\)
   D. \(3,200 \text{ m}^3\)
8. What is the height of a solid with a volume of 120 m\(^3\) and base area of 30 m\(^2\)? \(4 \text{ m}\)
9. Bradford has an aquarium with a base that is 22 inches \times 12 inches and a height that is 15 inches. What is the volume of the aquarium? Would the volume of the aquarium change if it did not have a lid? Explain.
   \(3,960 \text{ in}^3\); No; Sample answer: The dimensions remain the same even if one face is missing.
Combining Volumes

To find the volume of a solid made up of familiar parts, find the volume of each part and add the volumes.

Step 1: To find the volume of the figure at the right, separate the solid into two rectangular prisms. (See the dotted line in the figure.)

Step 2: Use the formula \( V = l \times w \times h \) to find the volume of each prism.

<table>
<thead>
<tr>
<th>Volume of Prism A</th>
<th>Volume of Prism B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V = 1 \times 4 \times 7 = 28 \text{ ft}^3 )</td>
<td>( V = 2 \times 4 \times 2 = 16 \text{ ft}^3 )</td>
</tr>
</tbody>
</table>

Step 3: Add the volumes of each prism. The volume of the solid is \( 28 + 16 = 44 \text{ ft}^3 \).

1. Show two ways of dividing the given solid into two rectangular solids.

2. Find the volume of the rectangular solid shown below. Show your work.

Sample answer: \( V = (4 \times 8 \times 3) + (3 \times 8 \times 4) = 96 + 96 = 192 \text{ cm}^3 \)

Practice Master

Combining Volumes

For 1 through 4, find the volume of each figure.

1. \( 243 \text{ in}^3 \)
2. \( 23 \text{ ft}^3 \)
3. \( 320 \text{ cm}^3 \)
4. \( 160 \text{ in}^3 \)

5. Paul wants to build the model with clay, but he does not know how many cubic centimeters of clay to purchase. How much clay should he purchase?
   A. \( 205 \text{ cm}^3 \)
   B. \( 355 \text{ cm}^3 \)
   C. \( 405 \text{ cm}^3 \)
   D. \( 935 \text{ cm}^3 \)

6. Ashley is stacking two boxes on a shelf. The bottom box measures 8 inches \( \times \) 5 inches \( \times \) 5 inches. The top box is a cube with one edge measuring 4 inches. What is the volume of this stack? Explain how you found your answer.

\( 214 \text{ in}^3 \); Add the volume of each box.

\( 150 + 64 = 214 \text{ in}^3 \).
Problem Solving: Use Objects and Reasoning

This cube has a volume of 1 cm³.

\[ V = 1 \times 1 \times 1 = 1 \text{ cm}^3 \]

\[ V = 2(1 \times 1 \times 1) = 2 \text{ cm}^3 \]

The same number of cubes will always have the same volume.

Each cube has a volume of 1 cm³.

1. Find the volume of the figure.

\[ V = 6 \text{ cm}^3 \]

2. Make and draw a figure of cubes that has a volume of 7 cm³.

3. Explain how you know how many cubes to use to draw the figure in Exercise 2.

The volume of each cube is 1 cm³, so 7 cm³ is equal to 7 cubes.

4. Find the volume.

\[ V = 5 \text{ cm}^3 \]

5. If the cubes in Exercise 4 were increased to 3 cm on a side, how would the volume be affected?

The volume would be changed by a factor of \(3 \times 3 \times 3\), or 27.

Find the volume of each figure of centimeter cubes.

1. \[ V = 5 \text{ cm}^3 \]
2. \[ V = 5 \text{ cm}^3 \]
3. \[ V = 9 \text{ cm}^3 \]
4. \[ V = 6 \text{ cm}^3 \]
5. \[ V = 6 \text{ cm}^3 \]
6. \[ V = 8 \text{ cm}^3 \]

7. Make and draw a figure of cubes that has a volume of 5 cm³.

8. Without building a model, tell whether a long row of 8 cubes or a cube made from 8 cubes would have a greater volume. Explain.

Both figures contain the same number of unit cubes, so both have the same volume.

9. Make and draw a figure that has the same volume as the diagram.

10. Find the volume of these figures. Then describe the pattern(s) you see. Can you determine the volume of the next figure in the pattern? Explain.

The volumes are the squares of the number of cubes on a side.

Descriptions of patterns will vary.