Adding and Subtracting Mixed Numbers

Dear Family,

In this topic, one of the concepts your child will be learning about is how to change a mixed number to an improper fraction and an improper fraction to a mixed number. A mixed number has a whole-number part and a fractional part, like 2 \( \frac{1}{3} \). An improper fraction has a numerator that is greater than or equal to the denominator, like \( \frac{3}{2} \).

To change 2 \( \frac{1}{3} \) to an improper fraction, multiply the denominator by the whole number. Add the numerator of the fraction. Write the new numerator over the same denominator:

\[
\begin{array}{c}
+ \\
2 \\
1 \\
\times \\
3 \\
\hline \\
7 \\
3 \\
\end{array}
\]

To change \( \frac{3}{2} \) to a mixed number, divide the numerator by the denominator. Then write the remainder over the divisor.

\[
\begin{array}{c}
4 \\
\underline{11} \\
3 \\
\hline \\
2 \\
3 \\
\end{array}
\]

To strengthen these skills your child can play the following game.

Fraction Frenzy

Materials: number cubes

1. Each player creates either a mixed number or an improper fraction by tossing the number cube three times. For example, if 1, 5, and 6 are tossed, \( \frac{15}{6} \) or \( \frac{10}{6} \) can be represented. The order of numbers tossed may need to be rearranged to form mixed numbers or improper fractions that can be changed.

2. Follow the steps shown above. Change the mixed numbers to improper fractions and the improper fractions to mixed numbers. The player who gets the correct answer first scores one point.
proper fraction
A fraction less than 1; its numerator is less than its denominator.

mixed number
A number that combines a whole number and a fraction.

improper fraction
A fraction in which the numerator is greater than or equal to its denominator.
Improper Fractions and Mixed Numbers

A mixed number combines a whole number with a fraction. It is greater than one.

An improper fraction has a numerator that is larger than its denominator.

How to Write an Improper Fraction as a Mixed Number

Write \( \frac{12}{5} \) as a mixed number.

Divide the numerator by the denominator.
The quotient is the whole number in the mixed number.
The remainder is the numerator.
The denominator stays the same.
\[
\frac{12}{5} = 2 \frac{2}{5}
\]

How to Write a Mixed Number as an Improper Fraction

Multiply the denominator by the whole number.
Then add the numerator.
Write this number for the numerator.
Use the original denominator.
\[
2 \frac{2}{5} = \frac{12}{5}
\]

1. Draw a picture to show \( \frac{3}{5} \).

For 2–4, write each improper fraction as a whole number or mixed number in simplest form.

2. \( \frac{30}{11} \)
3. \( \frac{36}{10} \)
4. \( \frac{2}{13} \)

Write each mixed number as an improper fraction.

5. \( 1 \frac{13}{10} \)
6. \( 6 \frac{2}{9} \)
7. \( 8 \frac{7}{8} \)

8. Write 6 as an improper fraction with a denominator of 10.

Check students’ drawings.

Write each improper fraction as a whole number or mixed number in simplest form.

9. \( \frac{79}{9} \)
10. \( \frac{52}{5} \)
11. \( \frac{5}{9} \)

Write each mixed number as an improper fraction.

12. \( \frac{23}{5} \)
13. \( \frac{59}{5} \)
14. \( \frac{77}{8} \)

Which letter on the number line corresponds to each number?

10. \( \frac{27}{5} \)
11. \( \frac{37}{10} \)
12. \( \frac{43}{8} \)

13. Which number does the model represent?

14. Can you express \( \frac{3}{5} \) as a mixed number? Why or why not?

No, \( \frac{3}{5} \) cannot be expressed only as a fraction or as a whole number (1).
Estimating Sums and Differences of Mixed Numbers

You can use rounding to estimate sums and differences of fractions and mixed numbers.

How to round fractions:
- If the fractional part is greater than or equal to 1/2, round up to the next whole number.
- Example: Round 31/2 to the nearest whole number.
  31/2 is greater than 1/2, so 31/2 rounds up to 4.
- If the fractional part is less than 1/2, drop the fraction and use the whole number you already have.
  Example: Round 6 1/2 to the nearest whole number.
  6 1/2 is less than 3/2, so drop 1/2 and round down to 6.

How to estimate sums and differences of fractions and mixed numbers:
- Round both numbers to the nearest whole number, then add or subtract.
- Examples:
  - Estimate 4 1/4 + 7 3/8:
    4 1/4 rounds down to 4.
    7 3/8 rounds up to 8.
    4 + 8 = 12
    So, 4 1/4 + 7 3/8 is about 12.

Round to the nearest whole number:

1. 8 2/3
2. 1 1 2/3
3. 4 1/2
4. 7 1/2
5. 5 2/3
6. 8 6/3

Estimate each sum or difference:

9. 7 1/2 + 6 1/2
10. 14 2/3 - 3 1/2
11. 2 1/4 + 3 1/2
12. 11 3/4 - 4 1/2
13. 9 + 3 1/2 + 8
14. 15 2/4 - 12 2/4

Practice Master

Estimating Sums and Differences of Mixed Numbers

Round to the nearest whole number.

1. 3 3/4
2. 6 1/3
3. 1 1/2
4. 12 3/10

Estimate each sum or difference:

5. 2 1/4 + 2 3/4
6. 3 1/3 - 1 1/3
7. 5 2/3 + 3 1/3
8. 11 - 6 1/2 + 2 3/4

Robert and May are competing in a track meet. The table at the right shows the results of their events.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Event</th>
<th>Results/Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robert</td>
<td>Long Jump</td>
<td>5 9/10 ft</td>
</tr>
<tr>
<td></td>
<td>Softball throw</td>
<td>62 2/3 ft</td>
</tr>
<tr>
<td>May</td>
<td>Long Jump</td>
<td>6 1/4 ft</td>
</tr>
<tr>
<td></td>
<td>Softball throw</td>
<td>7 3/4 ft</td>
</tr>
</tbody>
</table>

Yes, the difference is about 1 ft.

10. Use the table above. If the school record for the softball throw is 73 ft, about how much farther must Robert throw the ball to match the record?
   A 15 ft  B 16 ft  C 18 ft  D 20 ft

11. Consider the sum of 3 1/2. Round each fraction and estimate the sum. Add the two fractions using a common denominator and then round the result. Which estimate is closer to the actual answer?
   1 + 1 = 2; 12 3/20 + 15 2/20 = 27 3/20 = 1 7/20.
   This rounds to 1; the second estimate is closer to the actual answer.
**Reteaching Master**

**Modeling Addition and Subtraction of Mixed Numbers**

Example 1: Draw a model to add $1 \frac{1}{5} + 2 \frac{3}{4}$.

**Steps**
1. Model each mixed number using fraction strips.

2. Add the fractions. Regroup if you can.

3. Add the whole numbers to the regrouped fractions. Write the sum. Simplify, if possible.

Example 2: Draw a model to subtract $2 \frac{1}{5} - 1 \frac{3}{5}$.

**Steps**
1. Model the number you are subtracting from $2 \frac{1}{5}$.

2. Cross out one whole and $\frac{2}{5}$ to show subtracting $1 \frac{3}{5}$.

Express the part of the model that is not crossed out as a fraction or mixed number. So, $2 \frac{1}{5} - 1 \frac{3}{5} = \frac{3}{5}$.

Use fraction strips to find each sum or difference. Simplify, if possible.

1. $2 \frac{1}{2} + 1 \frac{1}{2}$
2. $2 \frac{6}{8} + 4 \frac{3}{8}$
3. $5 \frac{7}{8} + 3 \frac{5}{8}$
4. $2 \frac{2}{2} - 6 \frac{3}{4}$
5. $6 \frac{3}{4} - 3 \frac{2}{4}$
6. $8 \frac{3}{2} + 2 \frac{1}{2}$
7. $10 \frac{1}{2} - 5 \frac{1}{2}$
8. $8 \frac{3}{4} - 6 \frac{1}{4}$

**Practice Master**

**Modeling Addition and Subtraction of Mixed Numbers**

For 1 and 2, use each model to find each sum or difference.

1. $1 \frac{3}{8} + 1 \frac{7}{8}$
2. $3 \frac{1}{5} - 1 \frac{3}{5}$

Use fraction strips to find each sum or difference. Simplify, if possible.

3. $2 \frac{1}{3} + 2 \frac{3}{2}$
4. $3 \frac{1}{2} + 4 \frac{1}{2}$
5. $5 \frac{2}{7} - 1 \frac{1}{4}$
6. $1 \frac{2}{5} + 1 \frac{5}{7}$
7. $6 \frac{1}{3} - 3 \frac{2}{3}$
8. $4 \frac{5}{6} + 5 \frac{1}{2}$
9. $7 \frac{1}{4} - 4 \frac{3}{8}$
10. $6 \frac{2}{3} + 3 \frac{4}{3}$

11. $1 \frac{1}{3} + 3 \frac{2}{3}$
12. $2 \frac{1}{4} + 6 \frac{2}{3}$
13. $6 \frac{2}{9} - 4 \frac{2}{9}$
14. $5 \frac{1}{5} - 4 \frac{1}{5}$

15. Jerome’s rain gauge showed $12 \frac{5}{8}$ centimeters (cm) at the end of last month. At the end of this month, the rain gauge showed $15 \frac{3}{8}$ centimeters. How many more centimeters of rain fell this month?

A. $25 \frac{5}{8}$ cm
B. $15 \frac{5}{8}$ cm
C. $2 \frac{5}{8}$ cm
D. $1 \frac{5}{8}$ cm

16. You are adding $\frac{2}{3} + \frac{2}{3}$ using fraction strips. Explain how you rename the fraction part of the answer.

**Since $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$, I can rename $\frac{4}{3}$ as 1 whole strip and $\frac{1}{3}$ strip.**
Adding Mixed Numbers

1. \(\frac{3}{8} + \frac{1}{4}\)
2. \(\frac{1}{3} + \frac{3}{8}\)
3. \(\frac{5}{6} + \frac{1}{3}\)
4. \(\frac{1}{2} + \frac{5}{12}\)
5. \(\frac{2}{3} + \frac{7}{12}\)
6. \(\frac{3}{4} + \frac{2}{3}\)

Tirzah wants to put a fence around her garden. She has 22 yards of fence material. Does she have enough to go all the way around the garden?

No. She needs 22\(\frac{5}{6}\) yards to go around the garden.

Vital Organ Measures

<table>
<thead>
<tr>
<th></th>
<th>Average women's brain</th>
<th>Average man's brain</th>
<th>Average human heart</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11.9 kg</td>
<td>13.3 kg</td>
<td>8 kg</td>
</tr>
<tr>
<td></td>
<td>2.5 lb</td>
<td>3 lb</td>
<td>1.5 lb</td>
</tr>
</tbody>
</table>

9. What is the total weight of an average woman's brain and heart in pounds (lb)?

3.5 lb

10. What is the sum of the measures of an average man's brain and an average woman's brain in kilograms?

27.1 kg

11. Which is a good comparison of the estimated sum and the actual sum of \(\frac{3}{5} + \frac{2}{3}\)?

A. Estimated < actual
B. Estimated > actual
C. Actual < estimated
D. Actual = estimated

No; Sample answer: It is impossible for two mixed numbers to equal 2 because every mixed number is greater than 1.
**Subtracting Mixed Numbers**

The Plainville Zoo has had elephants for $1\frac{5}{4}$ years. The zoo has had zebras for $5\frac{1}{3}$ years. How many years longer has the zoo had elephants?

**Step 1:** Write equivalent fractions with the least common denominator. You can use fraction strips.

\[
\frac{5}{4} \quad \frac{1}{3}
\]

**Step 2:** Find the difference of $\frac{5}{4} - \frac{1}{3}$. Subtract the fractions. Then subtract the whole numbers. Simplify the difference if possible.

\[
9 \frac{5}{4} - 5 \frac{1}{3} = 4 \frac{1}{3}
\]

Example 3: Sometimes you may have to rename a fraction so you can subtract.

Find the difference of $\frac{6}{4} - \frac{2}{3}$.

\[
6 \frac{2}{4} \text{ rename } \frac{6}{4}
\]

For 1 through 4, find each difference. Simplify, if possible. Remember: You may have to rename a fraction in order to subtract.

1. $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$
2. $\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$
3. $\frac{3}{2} - \frac{1}{3} = \frac{1}{6}$
4. $\frac{6}{5} - \frac{2}{3} = \frac{1}{3}$

5. To find the difference of $7 - 3\frac{1}{2}$, how do you rename the 7?

I know that $\frac{12}{12} = 1$. I can rename 7 as $6\frac{12}{12}$.

6. Robyn ran $6\frac{2}{3}$ miles last week. She ran $4\frac{1}{2}$ miles this week. How many more miles did she run last week?

$1\frac{13}{20} = 1\frac{11}{20}$ more miles

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**Subtracting Mixed Numbers**

For 1 through 10, find each difference. Simplify, if possible.

1. $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$
2. $\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$
3. $\frac{3}{2} - \frac{1}{3} = \frac{1}{6}$
4. $\frac{6}{5} - \frac{2}{3} = \frac{1}{3}$

5. $9\frac{6}{8} - 6\frac{5}{8} = 2\frac{13}{18}$
6. $4\frac{2}{3} - 2\frac{1}{3} = 2\frac{1}{12}$
7. $8\frac{2}{3} - 3\frac{1}{3} = 2\frac{1}{12}$
8. $6\frac{1}{3} - 2\frac{3}{8} = 40$ in.
9. $\frac{5}{2} - \frac{2}{3} = \frac{5}{6}$
10. $2\frac{2}{10} - \frac{2}{3} = 1\frac{7}{30}$

**Egg Sizes in Inches (in.)**

<table>
<thead>
<tr>
<th>Bird</th>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada goose</td>
<td>$\frac{3}{2}$</td>
<td>$\frac{5}{12}$</td>
</tr>
<tr>
<td>Robin</td>
<td>$\frac{5}{2}$</td>
<td>$\frac{3}{8}$</td>
</tr>
<tr>
<td>Turtle dove</td>
<td>$\frac{11}{2}$</td>
<td>$\frac{9}{10}$</td>
</tr>
<tr>
<td>Raven</td>
<td>$\frac{1}{5}$</td>
<td>$\frac{3}{12}$</td>
</tr>
</tbody>
</table>

11. How much longer is the Canada goose egg than the raven egg? $\frac{1}{12}$ in. longer

12. How much wider is the turtle dove egg than the robin egg? $\frac{1}{2}$ in. wider

13. Which is the difference of $21\frac{13}{15} - 10\frac{7}{15}$?

A. $2\frac{2}{15}$  
B. $2\frac{3}{15}$  
C. $3\frac{13}{16}$  
D. $3\frac{3}{16}$

14. Explain why it is necessary to rename $\frac{3}{4}$ if you subtract $\frac{3}{4}$ from $\frac{1}{4}$, so you must borrow 1 whole from the 4 and rename $\frac{1}{4}$ as $\frac{4}{4}$. Sample answer: You cannot subtract $\frac{3}{4}$ from $\frac{1}{4}$.
More Adding and Subtracting Mixed Numbers

You can use what you know about adding and subtracting with mixed numbers when you simplify expressions with mixed numbers.

Simplify \( \frac{3}{4} + \frac{6}{5} \) - \( \frac{1}{2} \).

**Step 1** Add the mixed numbers in parentheses first. Find a common denominator.

\[
\frac{4}{5} + \frac{6}{7} = \frac{10}{3} - \frac{2}{3}.
\]

**Step 2** Subtract \( \frac{2}{3} \) from the sum you found. Find a common denominator.

\[
\frac{10}{3} - \frac{2}{3} = \frac{10}{3} - \frac{2}{3} = \frac{8}{3}.
\]

**Step 3** Rename if possible.

\[
\frac{9}{5} - \frac{3}{4} = \frac{7}{6}.
\]

In 1 through 8, simplify each expression. Remember to rename mixed numbers if possible.

1. \( \frac{4}{5} + \frac{6}{3} \) - \( \frac{2}{4} \)
2. \( \frac{6}{5} + \frac{2}{2} \) - \( \frac{3}{2} \)
3. \( \frac{10}{5} + \frac{2}{4} \) - \( \frac{2}{2} \)

4. \( \frac{5}{9} + \frac{1}{6} \) - \( \frac{2}{9} \)
5. \( \frac{1}{10} + \frac{1}{8} \) - \( \frac{1}{2} \)
6. \( \frac{2}{5} + \frac{1}{5} \) - \( \frac{1}{2} \)

7. \( \frac{1}{12} + \frac{1}{4} \) - \( \frac{1}{6} \)
8. \( \frac{1}{8} + \frac{3}{5} \) - \( \frac{1}{2} \)

9. \( \frac{5}{24} \) days

10. Suzy spent \( \frac{5}{2} \) days working on her English paper, \( \frac{3}{4} \) days doing her science project, and \( \frac{1}{2} \) days studying for her math test. How many days more did Suzy spend on her English paper and math test than on her science project?

Sample answer: 5 \( \frac{5}{24} \) days

11. Which shows three mixed numbers that have sum of 10?

A. \( \frac{1}{3} + \frac{5}{2} + \frac{2}{4} \)
B. \( \frac{1}{3} + \frac{5}{2} + \frac{2}{4} \)
C. \( \frac{1}{3} + \frac{5}{2} + \frac{2}{4} \)
D. \( \frac{1}{3} + \frac{5}{2} + \frac{2}{4} \)

12. What is a reasonable estimate for the sum of \( \frac{1}{3} + \frac{5}{2} + \frac{2}{4} \)?

Sample answer: \( 4 + \frac{3}{2} + \frac{5}{2} = 13 \)

16. Veronica is buying cubed cheese from Mr. Sand’s deli. She asks for \( \frac{1}{2} \) pounds. When Mr. Sand places some cheese in a container and weights it, the scale shows \( \frac{1}{2} \) pounds. The container weighs \( \frac{1}{2} \) pound. How many more pounds of cheese would Mr. Sand need to add to the scale to get the amount that Veronica asked for?

Sample answer: Subtract the weight of the container from the amount on the scale to find the weight of the cheese.

\( \frac{1}{4} - \frac{1}{6} = \frac{1}{3} \) pounds. Then subtract the difference from the amount asked for.

\( \frac{1}{4} - \frac{1}{6} = \frac{9}{16} \) pound.
Problem Solving: Draw a Picture and Write an Equation

A jeweler has a strand of gold wire that is 1\(\frac{3}{4}\) inches. He cuts \(\frac{3}{4}\) of an inch of wire to make a loop. How long is the remaining piece of wire?

**Read and Understand**

What do you know?
- The length of the wire is 1\(\frac{3}{4}\) inches.
- The length he cuts off is \(\frac{3}{4}\) of an inch.

What are you trying to find?
- The length of the wire that is left over.

**Plan and Solve**

Draw a picture for what you know.

Write an equation.
\[
\frac{3}{4} - \frac{3}{4} = x
\]

Solve the problem.
\[
\frac{3}{4} - \frac{3}{4} = \frac{1}{4}
\]

Write the answer in a sentence.
The remaining wire is \(\frac{1}{4}\) inch long.

**Look Back and Check**

Is your answer correct?
Yes, \(\frac{3}{4} - \frac{3}{4} = \frac{1}{4}\)

1. From his house, Jason rode his bike 1\(\frac{1}{2}\) miles to the post office. He then rode in the same direction to the park, which is \(\frac{1}{4}\) of a mile from the post office. How far did Jason ride?
   - 1. To the right, draw a picture to represent the problem to be solved. Let \(x\) = the distance Jason rode from his house to the park.
   
   \[
   x = \frac{3}{2} + \frac{1}{4} = \frac{7}{4} = 1\frac{3}{4}
   \]

2. Write an equation that represents this distance. Then solve for \(x\).
   \[
   \frac{11}{3} + \frac{1}{4} = x; \quad x = \frac{17}{12}
   \]

Practice Master

Name

Problem Solving: Draw a Picture and Write an Equation

Draw a picture and write an equation. Then solve.

1. Mr. Flanders drives 1\(\frac{1}{2}\) miles to school and 1\(\frac{1}{2}\) miles home each day. He also drives an extra 2\(\frac{1}{2}\) miles to go to the gym. How many miles does he drive in one day?

\[
\frac{12}{3} + \frac{12}{3} + \frac{5}{2} = x; \quad 2\frac{13}{21} \text{ miles}
\]

2. Allison is making a 16-inch necklace. The first 4\(\frac{3}{4}\) inches are filled with red beads and 9\(\frac{1}{2}\) inches are filled with blue beads. The rest has white beads. How many inches are filled with white beads?

\[
16 - \left(4\frac{3}{4} + 9\frac{1}{2}\right) = x; \quad x = 3\frac{3}{4} \text{ inches}
\]

3. Stewart draws a triangle, and each side is 2\(\frac{1}{2}\) inches long. Judith draws a square, and each side is 3\(\frac{3}{4}\) inches long. Which figure has the greater perimeter, the triangle or the square?

They both have the same perimeter of 6\(\frac{1}{2}\) inches.

4. Cristoff practices playing his guitar for 1\(\frac{1}{2}\) hours each weekday. He practices this amount of time plus an additional 1\(\frac{1}{2}\) hours on Sundays. Let \(x\) = the number of hours Cristoff practices on Sundays. Draw a picture and write an equation and solve to find the number of hours he practices on Sundays.

\[
1\frac{1}{2} + 1\frac{1}{2} = x; \quad x = 3 \text{ hours}
\]

5. Which of these fractions, when added to 2\(\frac{1}{2}\), will give you a sum greater than six?
   - A \(\frac{5}{3}\)
   - B \(\frac{3}{4}\)
   - C \(\frac{7}{3}\)
   - D \(\frac{7}{2}\)

6. Dennis says that 1\(\frac{1}{3}\), 1\(\frac{1}{2}\), and 1\(\frac{1}{4}\) are all equivalent. Is he correct? Draw a picture and explain your answer.

Yes; Check students’ work. It should show 3 sets of models for one whole plus half.